$\boxed{1:}$ Dirichlet's boundary conditions $u(t,a) = \alpha(t), u(t,b) = \beta(t)$, a_n, c_n disappear.
$\boxed{2:}$ Neumann boundary conditions $u_x(t,a) = 0, u_x(t,b) = 0$,$b_n, d_n$ dissapear.
$\boxed{3:}$ Mixed boundary conditions $u(t,a) = \alpha(t), u_x(t,b) = 0$
$\boxed{4:}$ Periodic boundary conditions $u(t,a) = u(t,b)\&u_x(t,a) = u_x(t,b)$.

$$\boxed{3}: u(t,x) = a_0 + c_0 t + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{nct\pi}{l}\right)\cos\left(\frac{n\pi x}{l}\right) + b_n \cos\left(\frac{nct\pi}{l}\right)\sin\left(\frac{n\pi x}{l}\right)\right.$$
$$\left. + c_n \sin\left(\frac{nct\pi}{l}\right)\cos\left(\frac{n\pi x}{l}\right) + d_n \sin\left(\frac{nct\pi}{l}\right)\sin\left(\frac{n\pi x}{l}\right)\right]$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{2\pi x}{l}\right)dx$$
$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{2\pi x}{l}\right)dx$$
$$c_n = \frac{2}{nc\pi}\int_0^l g(x)\cos\left(\frac{2\pi x}{l}\right)dx$$
$$d_n = \frac{2}{nc\pi}\int_0^l g(x)\sin\left(\frac{2\pi x}{l}\right)dx$$

Wave equation: $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, then

$$u(t,x) = \frac{f(x-ct)+f(x+ct)}{2} + \frac{1}{2c}\int_{x-ct}^{x+ct} g(z)dz$$

transport equation $\frac{\partial u}{\partial t} + c(x)\frac{\partial u}{\partial x} = 0$ then solve $\frac{dx}{dt} = c(x)$. for example $c(x) = e^{-x}$ then we get $e^x = t + C$. Let $\xi = e^x - t, \eta = e^x + t$ and $u(t,x) = v(\xi,\eta)$. Then differentiate, we see that solution only depends on ξ or η then use boundary conditions.

Heat equation: $u_t = u_{xx}$ so then use seperation of variables of $u(t,x) = T(t)X(x) \Rightarrow T'(t)X(x) = T(t)X"(x)$.

Fourier series: $f(x) = \sum_{n=0}^{\infty}[a_n\cos(nx) + b_n\sin(nx)]$ or $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ where

$$\begin{cases} a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)dx \\ b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)dx \\ c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-ikx}dx \end{cases}$$

For $Au_{tt} + Bu_{tx} + Cu_{xx} + Du_t + Eu_x + Fu = G$, then hyperbolic if $B^2 - 4AC > 0$, parabolic if $B^2 - 4AC = 0$ but $A^2 + B^2 + C^2 \neq 0$, elliptic if $B^2 - 4AC < 0$.